# MATH 2850: SEPARABLE EQUATIONS

**DEFINITION:** A DE is called **separable** if it can be written in the form: $h(y)\,y' = g(x)$ or $h(y)\,dy = g(x)\,dx$.

That is, we can 'separate' the variables on opposite sides of the equation.

**EXAMPLE:** Consider the DE: $y\,y' - x = 0$.

- Find a family of implicit solutions by separating variables.

Ans: $y^2 - x^2 = C$

- Find solutions (where possible) which satisfy the given ICs. State the corresponding interval of validity.

$y(5) = -4$

Ans: $y = -\sqrt{x^2 - 9}$ for $(3, \infty)$

$y(1) = 0$

Ans: No solution (why?)

$y(0) = 0$

Ans: $y = x$ and $y = -x$ for $(-\infty, \infty)$

**IMPLICIT FUNCTION THEOREM:** (Existence of solutions of separable equations.)

If $g$ is continuous on an open interval containing $x_0$ and $h$ is continuous on an open interval containing $y_0$ and, moreover, $h(y_0) \neq 0$, then there is a function $y = y(x)$ defined on an open interval containing $x_0$ such that $y$ is a solution to the IVP: $h(y)\, y' = g(x)$, $y(x_0) = y_0$.

**NOTE:** This guarantees existence of a solution - not necessarily uniqueness!

**EXAMPLE:** Which IVPs in the previous example satisfy the requirements of the Implicit Function Theorem?

**EXAMPLE:** Find an (implicit) solution to: $xy^2\, y' = y - x\, y'$, $y(1) = 3$.

Check your answer using implicit differentiation.

Ans: $y^2 + 2\ln(y) = 2\ln(3x) + 9$

**EXAMPLE:** The Logistic Equation: Solve $P'(t) = k\,P(t)\,(L - P(t))$, $P(0) = P_0$, $0 < P_0 < L$. Assume $k > 0$.

Ans: $P(t) = \dfrac{C\,L\,e^{kLt}}{1 + C\,e^{kLt}}$, $C = \dfrac{P_0}{L - P_0}$

**EXAMPLE:** Use the Logistic DE to show the solution $P(t)$ has an inflection point when $P(t) = \dfrac{L}{2}$.

**EXAMPLE:** Show the constant functions $P(t) = 0$ and $P(t) = L$ satisfy the Logistic DE.

What ICs correspond to these solutions?

**HOMEWORK:** pg. 52: 1-25 odd